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17EC54

## Fifth Semester B.E. Degree Examination, Aug./Sept. 2020 Information Theory \& Coding

Time: 3 hrs .

## Note: Answer any FIVE full questions, choosing ONE full question from each module.

## Module-1

1 a. Derive the expression for average information content of symbols in a long independent sequence.
(05 Marks)
b. A radio-jockey has a vocabulary of 10,000 words and he makes an announcement of 1000 words, selecting these words randomly from his vocabulary what is the information conveyed?
(05 Marks)
c. Consider the Markov source shown in Fig. Q1 (c). Find (i) State entropies
(ii) Entropy of the source (iii) $\mathrm{G}_{1}, \mathrm{G}_{2}$ and show that $\mathrm{G}_{1}>\mathrm{G}_{2}>\mathrm{H}(\mathrm{s})$.
(10 Marks)


2 a. Express Hartleys in bits and nats.
(04 Marks)
b. Obtain the entropies of the second and third extensions of a memoryless source emitting two symbols $\mathrm{n}_{1}$ and $\mathrm{n}_{2}$ with probabilities $\frac{1}{4}$ and $\frac{3}{4}$. Also show that $\mathrm{H}\left(\mathrm{S}^{2}\right)=2 \mathrm{H}(\mathrm{S}) \quad$ and $\mathrm{H}\left(\mathrm{S}^{3}\right)=3 \mathrm{H}(\mathrm{S})$.
(08 Marks)
c. For the Markov source shown below in Fig. Q2 (c), find (i) State probabilities (ii) State entropies (iii) Entropy of the Markov source.
(08 Marks)


Fig. Q2 (c)

## Module-2

3 a. Using Shannon's encoding algorithm encode the symbols A, B, C, D, E with probabilities $\frac{1}{8}, \frac{1}{16}, \frac{3}{16}, \frac{1}{4}$ and $\frac{3}{8}$. Find the coding efficiency and redundancy.
(06 Marks)
b. State kraft inequality. Find the smallest value of ' $r$ ' such that prefix codes can be constructed for the following code length requirements. $\mathrm{W}=\{1,4,4,4,5\}$ for corresponding $\mathrm{L}=\{1,2,3,4,5\}$. Also suggest a suitable code.
c. State and prove Shannon's source coding theorem.

## OR

4 a. Apply the Huffman's encoding procedure for the following set of symbols and hence determine the efficiency of the binary code so formed

| Symbol | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ |
| :--- | :---: | :---: | :---: |
| Probability | 0.7 | 0.15 | 0.15 |

If the same technique is applied to the second order extension of the above messages, what will be the improvement in efficiency?
(10 Marks)
b. A source emits 4 symbols $\left\{\mathrm{m}_{1}, \mathrm{~m}_{2}, \mathrm{~m}_{3}, \mathrm{~m}_{4}\right\}$ with probabilities $\left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8}\right\}$. Find the Shannon-Fano ternary code for the above symbols. Also find the efficiency and redundancy of coding.
(05 Marks)
c. Encode the following information using Lampel-Zir algorithm:

QUEUE_FOR_QUEEN'S_QUEST
(05 Marks)

## Module-3

5 a. For the JPM given below, compute $\overline{\mathrm{H}(\mathrm{X}), \mathrm{H}(\mathrm{Y}), \mathrm{H}(\mathrm{X}, \mathrm{Y}), \mathrm{H}(\mathrm{Y} / \mathrm{X}), \mathrm{H}(\mathrm{X} / \mathrm{Y}) \text { and } \mathrm{I}(\mathrm{X}, \mathrm{Y})}$

$$
\mathrm{P}(\mathrm{X}, \mathrm{Y})=\left[\begin{array}{cccc}
0.24 & 0 & 0.09 & 0.12 \\
0 & 0.16 & 0.12 & 0.09 \\
0.06 & 0.03 & 0 & 0.09
\end{array}\right]
$$

(08 Marks)
b. The noise matrix of a channel is as shown below find the capacity using Muraga's method. $\mathrm{P}(\mathrm{Y} / \mathrm{X})=\left[\begin{array}{ccc}3 / 4 & 1 / 4 & 0 \\ 0 & 1 / 2 & 1 / 2 \\ 1 / 3 & 0 & 2 / 3\end{array}\right]$.
(06 Marks)
c. Prove that $\mathrm{I}(\mathrm{A}, \mathrm{B})=\mathrm{I}(\mathrm{B}, \mathrm{A})$
(06 Marks)

OR
6 a. Two noisy channels are cascaded as shown below in Fig. Q6 (a). Find H(X), H(Y), H(Z), $\mathrm{H}(\mathrm{X}, \mathrm{Z}), \mathrm{H}(\mathrm{Z} / \mathrm{X})$ and $\mathrm{H}(\mathrm{X} / \mathrm{Z})$, given the probability of $\mathrm{p}\left(\mathrm{x}_{1}\right)=\mathrm{p}\left(\mathrm{x}_{2}\right)=0.5$
(08 Marks)

b. Derive the expression for maximum capacity of a Binary Erasure channel.
(07 Marks)
c. What are continuous channels? Write in brief the various entropies involved in continuous channels.
(05 Marks)

## Module-4

7 a. Consider a $(7,4)$ linear block code with the check bits defined as follows: $\mathrm{c}_{5}=\mathrm{d}_{1}+\mathrm{d}_{2}+\mathrm{d}_{3}$, $c_{6}=d_{2}+d_{3}+d_{4}$ and $c_{7}=d_{1}+d_{3}+d_{4}$. Write the Generator and Parity check matrices. Draw the circuit diagrams of the encoder and syndrome calculator. Also determine the error detection and correction capabilities of this code.
(10 Marks)
b. Design a feedback shift register encoder and syndrome calculator for a $(8,5)$ cyclic code with generator polynomial $g(x)=1+x+x^{2}+x^{3}$. Find the code vector for the message 11011 in systematic form. List all the states of the register and verify the value using the standard equation.
(10 Marks)

## OR

8
a. Explain in brief : (i) Hamming bound (ii) Linearity property
(iii) Minimum distance of a code.
(06 Marks)
b. Construct the standard array for a $(6,3)$ linear block code given, $\mathrm{P}=\left[\begin{array}{lll}1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$, where P is the Parity matrix. Detect and correct the errors for the received vectors $\mathrm{R}_{1}=100100$ and $\mathrm{R}_{2}=000011$.
(10 Marks)
c. Find the cyclic code in non-systematic format for the data vectors: (i) 1100 (ii) 1011 given $g(x)=1+x+x^{3}$.
(04 Marks)

## Module-5

9 a. Given a $(15,5)$ BCH code with generator polynomial $g(x)=1+x+x^{2}+x^{4}+x^{5}+x^{8}+x^{10}$, find the error detecting and correcting capabilities of this code.
(04 Marks)
b. Consider a $(3,1,2)$ convolution code with $\mathrm{g}^{(1)}=110, \mathrm{~g}^{(2)}=101, \mathrm{~g}^{(3)}=111$
(i) Draw the encoder block diagram.
(ii) Find the generator matrix and find the codeword corresponding to the information sequence 11101 using time domain approach.
(iii) Also verify the same using transform domain approach.
(10 Marks)
c. Write short notes on : (i) Code tree (ii) Trellis

## OR

10 a. Consider a $(2,1,2)$ convolutional encoder with $\mathrm{g}^{(1)}=111$ and $\mathrm{g}^{(2)}=101$. Draw the state diagram and Trellis for this encoder.
Also decode the code sequence $\{11,01,01,00,01,01,11\}$ using the Viterbi algorithm.
(12 Marks)
b. What are Golay codes? Explain.
c. Given a (3, 2, 1) convolutional encoder, define its (i) Constraint length (ii) Rate (iii) Draw the block diagram of the encoder. Given $\mathrm{g}_{1}^{(1)}=11, \mathrm{~g}_{1}^{(2)}=10, \mathrm{~g}_{1}^{(3)}=11$ and $\mathrm{g}_{2}{ }^{(1)}=01, \mathrm{~g}_{2}{ }^{(2)}=11, \mathrm{~g}_{2}{ }^{(3)}=00$ for data $\mathrm{d}_{1}=101$ and $\mathrm{d}_{2}=110$.
(04 Marks)

